On calculation of the off-shell renormalization functions in the R^2– gravity.

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Abstract

A new way how to calculate the off-shell renormalization functions within the R^2-gravity has been proposed. The one-loop renormalization group equations in the approach suggested have been constructed. The behaviour of effective potential for an massless scalar field interacting with the quantum gravitational field has been analyzed in this approach.

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As known, the Einstein theory of gravity is non-renormalizable [1]. Whereas the theory with terms which are quadratic in the curvature tensor is renormalizable in all orders of the perturbation theory [2] but not unitary [3] (ghosts and tachyons are present in its spectrum). So, this R^2-gravity cannot be accepted as a basic theory. Nevertheless, this theory may be considered as a model for studying the quantum gravitational effects. In particular, in the framework of this theory we can use the renormalization group method.

One of the problems in the R^2-gravity is related to the β_G-function calculation (G is the Newtonian constant). β_G calculated by a standard way is dependent on the gauge and parametrization [4]. It is commonly believed that this dependence can be explained if G is thought of as an inessential coupling constant which does not enter the renormalized S-matrix definition. However, we think of such a dependence as a flaw in the calculation method. One should remember that G is a quantity which can be measured in experiments and enters the classical gravitational potential definition [5]. The loop corrections to the gravitational potential are proportional to the β_G-function. It leads to the gauge and parametrization dependence of physical quantities. Hence, in our opinion, the β_G-function should not depend on the gauge and parametrization. For getting the β_G-function which is independent on the gauge and parametrization we should base on some additional suggestions in the framework of the traditional procedure or should calculate new objects like the Vilkovisky-DeWitt effective action [6]. However, in the Vilkovisky-DeWitt formalism there is an ambiguity in the choice of the configurational space metric for the quantum gravity. For this reason β_G will depend on this metric. In this paper we suggest a new way for this problem solution. The method suggested for finding the correct β_G-function is based on putting the non-zero renormalization constant on the metric field [3].

Let us consider the R^2-gravity with Lagrangian [4]

[3]The non-zero renormalization constant for the metric field in $(2+\varepsilon)$-gravity has been considered in refs [7].

[4]We use the following notations:

$$c = \hbar = 1; \qquad \mu,\nu = 0,1,2,3; \qquad (g) = det(g_{\mu\nu}), \qquad \varepsilon = \frac{4-d}{2}$$

$$R^{\sigma}{}_{\lambda\mu\nu} = \partial_\mu \Gamma^{\sigma}{}_{\lambda\nu} - \partial_\nu \Gamma^{\sigma}{}_{\lambda\mu} + \Gamma^{\sigma}{}_{\alpha\mu}\Gamma^{\alpha}{}_{\lambda\nu} - \Gamma^{\sigma}{}_{\alpha\nu}\Gamma^{\alpha}{}_{\lambda\mu}, \qquad R_{\mu\nu} = R^{\sigma}{}_{\mu\sigma\nu}, \qquad R = R_{\ \mu\nu} g^{\mu\nu}$$

where $\Gamma^{\sigma}{}_{\mu\nu}$ is the Riemann connection and $W^2 = R^2_{\mu\nu} - \frac{1}{3}R^2, \nabla^2 = g^{\mu\nu}\nabla_\mu\nabla_\nu$

$$L_{GR} = \Big(\frac{1}{\lambda}W^2 - \frac{\omega}{3\lambda}R^2 - \frac{R}{k^2} + \frac{2\sigma}{k^4}\Big)\sqrt{-g} \qquad (1)$$

where λ, ω and σ are the dimensionless constants, $k^2 = 16\pi G$. The space-time we work is topologically trivial, without a boundary, the Euler number equals zero. Thus, we have a right to use the relation: $R^2_{\mu\nu\sigma\lambda} = 4R^2_{\mu\nu} - R^2$.

The theory described by the action (1) is multiplicatively renormalizable in all orders of the perturbation theory. The calculation of Green functions as well as S-matrix elements including radiative corrections can be carried out in the framework of the theory under consideration. For getting the finite Green functions in the standard field theory not only physical parameters but fields themselves also should be renormalized. As to the S-matrix elements the result has to be the same as when fields are not renormalized. Following the arguments of [8] it can be shown that the renormalization of quantum and ghost fields in the background field method is not essential to the one-loop background Green functions calculation. Consequently, at the one-loop level, in the theory (1) the five renormalization constants may arise: for the physical parameters $Z_\lambda, Z_\omega, Z_G, Z_\sigma$ and for the background metric field Z_g. All one-loop singularities of the theory can be absorbed into these constants. In the MS-scheme all Z_i constants contain only poles in ε.

Let us consider how the one-loop renormalization group equations will be modified due to the non-zero renormalization constant for the metric field. Let the tensor density $g^*_{\mu\nu} = g_{\mu\nu}(-g)^r$ (where $r \neq -1/4$) play the role of a dynamical variable. The one-loop counterterms in the background field method have the form: [5]

$$\Gamma^{GR}_{div} = \frac{1}{\varepsilon}\int \left(\theta_2 W^2 + \frac{\theta_3}{3}R^2 + \theta_4\frac{R}{k^2} + \frac{\theta_5}{k^4}\right)\sqrt{-g}\ d^4x. \qquad (2)$$

[5]For the sake of completeness we present the results $\{\theta_i\}$ in the trivial parametrization $r = 0$ and in the minimal gauge given in the paper [6]

$$\theta_2 = \frac{1}{16\pi^2}\frac{133}{20}, \qquad \theta_3 = \frac{1}{16\pi^2}\Big(\frac{5}{3}\omega^2 + \frac{5}{2}\omega + \frac{5}{24}\Big), \qquad \theta_4 = \frac{1}{16\pi^2}\Big(\frac{5}{3}\omega - \frac{13}{12} - \frac{1}{8\omega}\Big)\lambda,$$

$$\theta_5 = \frac{1}{16\pi^2}\Big(\frac{5}{4}\lambda^2 + \frac{\lambda^2}{16\omega^2} + \frac{28}{3}\sigma\lambda + \frac{1}{3}\frac{\sigma\lambda}{\omega}\Big)$$

Supposing that after multiplicative redefinition of the metric and parameters

$$\begin{array}{rcl} g^{*}_{\mu\nu} & \to & g^{*B}_{\mu\nu} = Z_g g^{*R}_{\mu\nu} = (1+\delta Z_g)\, g^{*R}_{\mu\nu}, \\ \lambda & \to & \lambda_B = Z_\lambda \lambda_R = (\lambda_R + \delta Z_\lambda)\,, \\ \omega & \to & \omega_B = Z_\omega \omega_R = (\omega_R + \delta Z_\omega)\,, \\ \sigma & \to & \sigma_B = Z_\sigma \sigma_R = (\sigma_R + \delta Z_\sigma)\,, \\ G & \to & G_B = Z_G G_R = (1+\delta Z_G)\, G_R \end{array} \tag{3}$$

the one-loop background Green functions obtained from the effective action $\Gamma_R = \Gamma^{GR}_B - \Gamma^{GR}_{div}$ should be finite at $\varepsilon \to 0$ we have the following system of equations for δZ_i

$$\delta Z_\lambda = -\frac{\theta_2}{\varepsilon}\lambda_R^2, \tag{4}$$

$$\delta Z_\omega = -\frac{\theta_3 + \theta_2\omega_R}{\varepsilon}\lambda_R, \tag{5}$$

$$\delta Z_\sigma = \frac{1}{2}\frac{\theta_5 + 4\theta_4\sigma_R}{\varepsilon}, \tag{6}$$

$$\delta Z_G - \frac{1}{s}\delta Z_g = \frac{\theta_4}{\varepsilon} \tag{7}$$

where $s \equiv 4r + 1 \neq 0$ and we take into account that

$$g^{\mu\nu} \to \left(1 - \frac{1}{s}\delta Z_g\right) g^{\mu\nu}, \qquad \sqrt{g} \to \left(1 + \frac{2}{s}\delta Z_g\right)\sqrt{g}.$$

The renormalization constant for the metric field appears only in the terms $R\sqrt{-g}/k^2$ and $\sqrt{-g}/k^4$. It is conditioned by the fact that the tensor $R^{\sigma}{}_{\mu\lambda\nu}$ is built out of the combinations $\phi^{-1}\partial\phi$ and is invariant under multiplicative redefinition of the fields. Moreover, at the same time the combination $g^{\mu\nu}g^{\alpha\beta}\sqrt{-g}$ is invariant under multiplicative redefinition of the metric. So, in the gravity compared with the standard field theory the renormalization constant for the field is defined by low powers of the kinetic term.

In order to find a definite solution for the equation (7) we need some additional conditions. We don't know exactly how to find new equation. We suggest that such an additional equation can be obtained in the on-shell

approach. We suppose that all on-shell divergences can be removed by the redefinition of only physical parameters (the gravitational and dimensionless constants). This assumption is based on two points:

- S-matrix in the background field method is identical to the conventional S-matrix [9]
- for the renormalized S-matrix elements calculation we should renormalize only physical parameters.

As known, for the R^2–gravity we may limit ourselves to using only the trace of the motion equation [10]

$$g_B^{*\mu\nu}\frac{\delta L_{GR}}{\delta g_B^{*\mu\nu}} = \left(-\frac{s}{k_B^2}\Big(R_B - \frac{4\sigma_B}{k_B^2}\Big) + 2s\frac{\omega_B}{\lambda_B}\nabla^2 R_B\right)\sqrt{-g_B} \equiv 0. \tag{8}$$

Taking into account (4–8), we have

$$\delta Z_G = 0, \tag{9}$$

$$\delta Z_g = -s\frac{\theta_4}{\varepsilon}. \tag{10}$$

It is easy to show that in our approach $\delta Z_G = 0$ in all orders of the perturbation theory. The one-loop counterterms are not polynoms in Lagrangian parameters (see the footnote 5 at the page 2). It means the smallness of δZ_i is provided by the expansion over constants $\hbar$ (loop expansion). In this case in order to calculate generalized β-functions we can use the equations ¿from [11]. Introducing the definition $\mu^2\frac{d}{d\mu^2}g_{Bare} = -\varepsilon g + \beta_g$, we obtain that in the MS-scheme at the one-loop level

$$\beta_i = \delta Z_i \varepsilon. \tag{11}$$

At the one-loop level the coefficients θ_2, θ_3 and the combination $\theta_5 + 4\sigma_R\theta_4$ are independent on the gauge and parametrization off-shell [12]. Using the relations (4) - (6), (9) - (11), we get that the one-loop β–functions for physical parameters are also independent on the gauge and parametrization. $\beta_G = 0$ in all orders of the perturbation theory. So, in the space without boundaries and interactions with the matter fields there are no loop corrections to the

Newtonian constant. As to the coefficient θ_4 and the one-loop anomalous dimension of the metric field γ_g they depend on the gauge and parametrization [4]. It does not contradict to the basics of the quantum theory. The introduction of the non-zero renormalization constant for the metric field allows also to explain the dependence of one-loop results from the configurational space metric in the Vilkovisky-DeWitt formalism. The standard form for the configurational space metric in the gravity is

$$\gamma^{\mu\nu\alpha\beta} = \left(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - \mathbf{a}g^{\mu\nu}g^{\alpha\beta}\right)\sqrt{-g}$$

In order to fix **a** it is required [13] that the metric should be fixed in the space of fields in accordance with the classical action coefficients at the highest space-time derivatives. The calculations performed for the quantum gravity in the Vilkovisky-DeWitt formalism lead to a dependence of physical quantities from **a** [14]. At present this question (correct choice of the configurational space metric in the gravity) is still open. By using the method for the renormalization functions calculation suggested in this work within the Vilkovisky-DeWitt formalism, we have $\beta_G = 0$ and all the dependence from **a** is absorbed into the anomalous dimension of the metric field γ_g. This dependence of γ_g from **a** can be explained in precisely the same way as the gauge and parametrization dependence of the field anomalous dimension in the standard quantum field theory.

It can be manifested that the multiplicative renormalization of the metric field is related to only the conformal mode. The easiest way to do it is to choose a conformal parametrization where dynamical variables are $\psi_{\mu\nu} = g_{\mu\nu}(-g)^{-\frac{1}{4}}$ and $\pi = (-g)^{\frac{m}{4}}$, $m \neq 0$, $\; det\psi_{\mu\nu} = 1$. In this parametrization the fields $\psi_{\mu\nu}$ and π are considered to be independent dynamical variables. As a consequence, in the general case there are two different renormalization constants Z_ψ, Z_π for the fields ψ and π respectively. The similar arguments we have used above result in the following system for the one-loop renormalization constants δZ_i definition

$$
\begin{aligned}
-\frac{\delta Z_\lambda}{\lambda^2} - 2\delta Z_\psi &= \frac{\theta_2}{\varepsilon}, \\
\frac{\delta Z_\omega}{\lambda} - \frac{\omega}{\lambda^2}\delta Z_\lambda - 2\frac{\omega}{\lambda}\delta Z_\psi &= \frac{\theta_3}{\varepsilon}, \\
\delta Z_\sigma - 2\left(\delta Z_G - \frac{1}{m}\delta Z_\pi\right)\sigma_R &= \frac{1}{2}\frac{\theta_5}{\varepsilon}, \\
\delta Z_G + \delta Z_\psi - \frac{1}{m}\delta Z_\pi &= \frac{\theta_4}{\varepsilon}
\end{aligned}
$$

where we use the relations $\psi^B_{\mu\nu} = (1+\delta Z_\psi)\,\psi^R_{\mu\nu}$ and $\pi_B = (1+\delta Z_\pi)\,\pi_R$. The conditions of renormalizability on-shell give some additional equations for the renormalization constants definition. As a result, the old solutions (4)–(6), (9) are obtained once more, the solution (10) will be replaced with

$$
\delta Z_\psi = 0, \tag{12}
$$

$$
\delta Z_\pi = -m\frac{\theta_4}{\varepsilon}. \tag{13}
$$

Let us consider the R^2–gravity interacting with an massless scalar field. The gravitational field Lagrangian has the form (1), the Lagrangian for the scalar field is

$$
L_{mat} = \left(\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{\rho\varphi^4}{4!} + \frac{1}{2}\xi R\varphi^2\right)\sqrt{-g} \tag{14}
$$

where ρ, ξ are the dimensionless constants. Let the tensor density $g^*_{\mu\nu} = g_{\mu\nu}(-g)^r$, where $r \neq -1/4$ and scalar density $\phi = \varphi(-g)^\chi, \chi$ is an arbitrary number be dynamical quantities. The one-loop divergencies in the background field method are $\Gamma_{div} = \Gamma^{GR}_{div} + \Gamma^{mat}_{div}$, where the functional form of Γ^{GR}_{div} is given in (2), and

$$
\Gamma^{mat}_{div} = \frac{1}{\varepsilon}\int\left(\gamma_1\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \gamma_2\frac{\varphi^4}{4!} + \gamma_3\frac{1}{2}R\varphi^2\right)\sqrt{-g}\ d^4x. \tag{15}
$$

Supposing that all divergencies can be removed by the renormalization of constants and fields, the following equations system for the renormalization

constants is obtained in addition to (4)–(6), (9), (10) [6]

$$\begin{aligned}
\delta Z_\xi &= \frac{\gamma_1 \xi - \gamma_3}{\varepsilon}, && (16)\\
\delta Z_\rho &= \frac{2\gamma_1 \rho - \gamma_2}{\varepsilon}, && (17)\\
\delta Z_\phi &= -\frac{1}{2}\frac{\gamma_1}{\varepsilon} - \frac{1-8\chi}{2s}\delta Z_g. && (18)
\end{aligned}$$

where $\phi_B = (1+\delta Z_\phi)\,\phi_R$, $\xi_B = (\xi_R + \delta Z_\xi)$ and $\rho_B = (\rho_R + \delta Z_\rho)$. In the conformal parametrization the relations (17), (18) should be replaced with the following

$$\begin{aligned}
\delta Z_\rho &= \frac{2\gamma_1 \rho - \gamma_2}{\varepsilon} - 2\rho\delta Z_\psi,\\
\delta Z_\phi &= -\frac{1}{2}\frac{\gamma_1}{\varepsilon} - \frac{1-8\chi}{2m}\delta Z_\pi + \frac{1}{2}\delta Z_\psi
\end{aligned}$$

where δZ_ψ and δZ_π are defined in (12) and (13) respectively. The renormalization constant for the scalar field is related to the corresponding constant for the metric field if $\chi \neq 1/8$. So, in our approach the renormalization group equation for the scalar field constant change its form, and the non-zero renormalization constant for the metric field appears.

Let us analyze the renormalization group for the improved effective potential [15]. The renormalization group equation for the effective potential in our approach has the following form

$$\left(\mu^2\frac{\partial}{\partial\mu^2} + \beta_\rho\frac{\partial}{\partial\rho} + \beta_\xi\frac{\partial}{\partial\xi} - \gamma_\phi\phi\frac{\delta}{\delta\phi} + \gamma_g g^{*\mu\nu}\frac{\delta}{\delta g^{*\mu\nu}}\right)V = 0 \qquad (19)$$

where $\beta_\rho, \beta_\xi, \gamma_\phi$ and γ_g are the renormalization group functions; ϕ is the scalar density and $g^{*\mu\nu}$ is the tensor density respectively. We consider $R_{\mu\nu}$ as

[6]For this theory one needs to use the following equation

$$g_B^{*\mu\nu}\frac{\delta L}{\delta g_B^{*\mu\nu}} = \left(-\frac{s}{k_B^2}\left(R_B - \frac{4\sigma_B}{k_B^2}\right) + 2s\nabla^2\left(\frac{\omega_B}{\lambda_B}R_B + \frac{6\xi-1}{8}\varphi_B^2\right)\right)\sqrt{-g_B} \equiv 0.$$

As a consequence, the eqs. (9) and (10) do not change their functional form.

a kinetic term. As a consequence, the derivative $\frac{\delta}{\delta g^{*\mu\nu}}$ does not influence on $R_{\mu\nu}$, but only on $g^{\mu\nu}\sqrt{-g}$ and $\sqrt{-g}$. Splitting the potential V into two parts (V_1 which is independent on the curvature and V_2 linear in the curvature), we obtain two equations

$$\left(\mu^2\frac{\partial}{\partial\mu^2}+\beta_\rho\frac{\partial}{\partial\rho}-4\gamma_\phi-2\frac{1-8\chi}{s}\gamma_g\right)V_1=0 \tag{20}$$

$$\left(\mu^2\frac{\partial}{\partial\mu^2}+\beta_\xi\frac{\partial}{\partial\xi}-2\gamma_\phi-\frac{1-8\chi}{s}\gamma_g\right)V_2=0 \tag{21}$$

The solutions of eqs. (20) and (21) are the following:

$$V(t)=\frac{\rho(t)\varphi^4}{4!}\sqrt{-g}f^4(t)-\frac{1}{2}\xi(t)R\varphi^2\sqrt{-g}f^2(t) \tag{22}$$

where

$$t = \ln\frac{\varphi^2(-g)^{2\chi_1}}{\mu^2} \tag{23}$$

$$f(t) = \exp\left[-\int_0^t\frac{\gamma_\phi(\tau)+\frac{1-8\chi}{2s}\gamma_g(\tau)}{1+2\gamma_\phi(\tau)+8\frac{\chi_1-\chi}{s}\gamma_g(\tau)}d\tau\right] \tag{24}$$

$$\frac{d\rho(t)}{dt} = \frac{\beta_\rho(t)}{1+2\gamma_\phi(t)+8\frac{\chi_1-\chi}{s}\gamma_g(t)}, \qquad \rho(0)=\rho \tag{25}$$

$$\frac{d\xi(t)}{dt} = \frac{\beta_\xi(t)}{1+2\gamma_\phi(t)+8\frac{\chi_1-\chi}{s}\gamma_g(t)}, \qquad \xi(0)=\xi \tag{26}$$

and we use the following initial condition

$$V(t=0)=V_{cl}\equiv\left(\frac{\rho\varphi^4}{4!}-\frac{1}{2}\xi R\varphi^2\right)\sqrt{-g} \tag{27}$$

If $\chi_1\neq 1/8$ then the solution (22) differs from a standard one [15]. However, in the one-loop approximation where $f(t)=1-\left(\gamma_\phi+\frac{1-8\chi}{2s}\gamma_g\right)t$ and $\rho(t)=\rho+\beta_\rho t, \xi(t)=\xi+\beta_\xi t$, the effective potential (22) is the same as the standard one at arbitrary χ and χ_1. It can be explained by the fact that for the field combinations $\varphi^4\sqrt{-g}$ and $\varphi^2 g^{\mu\nu}\sqrt{-g}$ entering the effective potential

the one-loop renormalization equation in our approach (see eq. (18)) has the same form as the traditional one [15].

In the conformal parametrization the renormalization group equation for the effective potential is

$$\left(\mu^2\frac{\partial}{\partial\mu^2}+\beta_\rho\frac{\partial}{\partial\rho}+\beta_\xi\frac{\partial}{\partial\xi}-\gamma_\phi\phi\frac{\delta}{\delta\phi}-\gamma_\pi\pi\frac{\delta}{\delta\pi}+\gamma_\psi\psi^{\mu\nu}\frac{\delta}{\delta\psi^{\mu\nu}}\right)V=0 \qquad (28)$$

The functional form for this solution is the same as for (22), where we use the condition $\gamma_\psi = 0$ (see eq.(12)). The definition of t and the initial conditions are given in (23) and (27). And now

$$f(t) = \exp\left[-\int_0^t \frac{\gamma_\phi(\tau)+\frac{1-8\chi}{2m}\gamma_\pi(\tau)}{1+2\gamma_\phi(\tau)+8\frac{\chi_1-\chi}{m}\gamma_\pi(\tau)}d\tau\right] \qquad (29)$$

$$\frac{d\rho(t)}{dt} = \frac{\beta_\rho(t)}{1+2\gamma_\phi(t)+8\frac{\chi_1-\chi}{m}\gamma_\pi(t)}, \qquad \rho(0)=\rho \qquad (30)$$

$$\frac{d\xi(t)}{dt} = \frac{\beta_\xi(t)}{1+2\gamma_\phi(t)+8\frac{\chi_1-\chi}{m}\gamma_\pi(t)}, \qquad \xi(0)=\xi \qquad (31)$$

This paper shows that in the quantum $4D$ R^2–gravity the non-zero renormalization constant for the metric field may exist and depends on the choice of gauge and parametrization. Only the conformal metric mode should be renormalized. In the gravity compared with the standard field theory the renormalization constant for the field is defined by low powers of the kinetic term. The β–function for physical parameters obtained in this approach is independent on the gauge and parametrization at the one-loop level in the MS-scheme in the dimensional regularization. The β-function for the Newtonian constant equals zero in the space-time without a boundary in all order of the perturbation theory both without and with massless fields interactions. The renormalization constants for matter fields acquire a dependence on the metric field renormalization. In spite of changes in the renormalization group equations the effective potential for an arbitrary massless theory at the one-loop level in the linear curvature approximation remains invariable and is the same as in [15].

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